Chapter 11

Inductors
Objectives

- Describe the **basic structure** and **characteristics** of an inductor
- Discuss various types of inductors
- Analyze **series** inductors
- Analyze **parallel** inductors
- Analyze inductive **dc** switching circuits
- Analyze inductive **ac** circuits
The Basic Inductor

- When a length of wire is formed onto a coil, it becomes a basic **inductor**

- When there is current through the coil, a three-dimensional **electromagnetic field** is created, surrounding the coil in all directions
Basics of a Inductance

The unit of inductance is the **henry** (H), defined as the inductance when one ampere per second through the coil, induces one volt across the coil

\[ v(t) = L \frac{di(t)}{dt} \]

In many practical applications, mH (10^{-3} H) or μH (10^{-6} H) are the more common units.
Basics of a Inductance

**Inductance** is a measure of a coil’s ability to establish an induced voltage as a result of a change in its current, and that induced voltage is in a direction to oppose that change in current.

An inductor stores **energy** in the magnetic field created by the current: \( U = \frac{1}{2} LI^2 \) (J)
Physical Characteristics

- Inductance is directly proportional to the \textit{permeability} of the core material.
- Inductance is directly proportional to the cross-sectional area of the core.
- Inductance is directly proportional to the square of the \textbf{number of turns} of wire.
- Inductance is inversely proportional to the length of the core material.

\[ L = \mu \frac{N^2 A}{l} \quad \text{(H)} \]
Physical Characteristics

Example: Determine the inductance of the coil

\[ A = \pi r^2 = \pi (0.25 \times 10^{-2})^2 = 1.96 \times 10^{-5} \text{ m}^2 \]

\[ L = \mu N^2 A/l = (0.25 \times 10^{-3})(350)^2(1.96 \times 10^{-5})/(0.015) = 40 \text{ mH} \]
Winding Resistance

- When many turns of wire are used to construct a coil, the total resistance may be significant.
- The inherent resistance is called the dc resistance or the winding resistance ($R_W$).

(a) The wire has resistance distributed along its length.

(b) Equivalent circuit
Winding Capacitance

- When two conductors are placed side-by-side, there is always some capacitance between them.
- When many turns of wire are placed close together in a coil, there is a **winding capacitance** \(C_W\).
- \(C_W\) becomes significant at high frequencies.

(a) Stray capacitance between each loop appears as a total parallel capacitance \(C_W\).

(b) Equivalent circuit
Faraday’s Laws

Faraday’s law:

- The amount of voltage induced in a coil is directly proportional to the rate of change of the magnetic field with respect to the coil
Lenz’s Laws

Example of Lenz’s law:

(a) Switch open: Constant current and constant magnetic field; no induced voltage.

(b) At instant of switch closure: Expanding magnetic field induces voltage, which opposes an increase in total current. The total current remains the same at this instant.

(c) Right after switch closure: The rate of expansion of the magnetic field decreases, allowing the current to increase exponentially as induced voltage decreases.

(d) Switch remains closed: Current and magnetic field reach constant value.
**Lenz’s Laws**

- **Example of Lenz’s law:**

  (a) Switch open: Constant current and constant magnetic field; no induced voltage.

  (b) At instant of switch opening: Magnetic field begins to collapse, creating an induced voltage, which opposes a decrease in current.

  (f) After switch opening: Rate of collapse of magnetic field decreases, allowing current to decrease exponentially back to original value.
Typical Inductors

(a) Fixed

(b) Variable
Series Inductors

When inductors are connected in series, the total inductance increases

\[ L_T = L_1 + L_2 + L_3 + \ldots + L_n \]
Parallel Inductors

When inductors are connected in parallel, the total inductance is less than the smallest inductance:

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots + \frac{1}{L_n}$$
Inductors in DC Circuits

- When there is constant current in an inductor, there is **no induced voltage**
- There is a voltage drop in the circuit due to the winding resistance of the coil
- Inductance itself appears as a **short** to dc

\[ V = 0 \]

\[ P = I^2 R_w \]

Conversion of electrical energy to heat due to winding resistance

Energy stored in magnetic field

\[ W = \frac{1}{2} LI^2 \]
Inductors in DC Circuits

Illustration of the exponential buildup of current in an inductor
Inductors in DC Circuits

Illustration of the exponential decrease of current in an inductor

(a) Initially, there is 1 A of constant current. Then SW2 is closed and SW1 opened simultaneously ($t = 0$).

(b) At the end of one time constant ($t = \tau$)

(c) At the end of two time constants ($t = 2\tau$)

(d) At the end of three time constants ($t = 3\tau$)

(e) At the end of four time constants ($t = 4\tau$)

(f) At the end of five time constants ($t = 5\tau$). Since only 1% of the current is left, this value is taken as the final zero value.
Because the inductor’s basic action opposes a change in its current, it follows that current cannot change instantaneously in an inductor.

The rate, a certain time is required for the current to make a change from one value to another, is determined by the **RL time constant** ($\tau = L/R$).
Energizing Current in an Inductor

- In a series RL circuit, the current will increase to approximately 63% of its full value in **one time-constant** \((\tau)\) interval after the switch is closed.
- The current reaches its final value in approximately \(5\tau\).

\[
i = I_F \left(1 - e^{-tR/L}\right)
\]
De-energizing Current in an Inductor

- In a series RL circuit, the current will decrease to approximately 63% of its fully charged value one time-constant ($\tau$) interval after the switch is closed.
- The current reaches 1% of its initial value in approximately $5\tau$; considered to be equal to 0.

\[ i = I_i e^{-tR/L} \]
RL Time Constant

Example: Determine the inductor current 30 μs after the switch is closed

\[ \tau = \frac{L}{R} = \frac{100 \text{mH}}{2.2 \text{kΩ}} = 45.5 \text{ μs} \]

\[ I_F = \frac{V_s}{R} = \frac{12 \text{V}}{2.2 \text{kΩ}} = 5.45 \text{ mA} \]

\[ i_L = I_F (1 - e^{-\frac{Rt}{L}}) = (5.45)(1 - e^{-0.66}) = 2.63 \text{ mA} \]
RL Time Constant

Example: Determine the inductor current 2 ms after the switch(1) is opened and switch(2) is closed

\[ \tau = \frac{L}{R} = \frac{200 \text{mH}}{56 \Omega} = 3.57 \text{ ms} \]

\[ I_i = \frac{V_s}{R} = \frac{5 \text{V}}{56 \Omega} = 89.3 \text{ mA} \]

\[ i_L = I_i (e^{-\frac{Rt}{L}}) = (89.3)(e^{-0.56}) = 51.0 \text{ mA} \]
Analysis of Inductive AC Circuit

The current lags the voltage by 90° in a purely inductive ac circuit

\[ v(t) = L \frac{di(t)}{dt} \]
Ohm’s Law in Inductive AC Circuits

The reactance \( (X_L) \) of a inductor is analogous to the resistance \( (R) \) of a resistor

\[
V = IZ_L = I(j \omega L) = I(jX_L)
\]

**Magnitude** → \( V = IX_L \)

\( V = IR \)
Inductive Reactance, $X_L$

The relationship between inductive reactance, inductance and frequency is:

$$X_L = 2\pi f L \ (\Omega)$$

Example: Determine the inductive reactance

$$X_L = 2\pi f L$$
$$= 2\pi (1 \times 10^3) (5 \times 10^{-3})$$
$$= 31.4 \ \text{k}\Omega$$
Inductive Reactance, $X_L$

\[ I = \frac{V}{X_L} = \frac{V}{2\pi f L} \]
Inductive Reactance, $X_L$

$$I = \frac{V}{X_L} = \frac{V}{2\pi f L}$$

(a) Less inductance, more current.

(b) More inductance, less current.
Inductive Reactance, $X_L$

Example: Determine the rms current in below circuit

\[ X_L = 2\pi(10\times10^3)(100\times10^{-3}) = 6283 \ \Omega \]
\[ V_{\text{rms}} = I_{\text{rms}}X_L \]
\[ I_{\text{rms}} = \frac{V_{\text{rms}}}{X_L} = \frac{(5V)}{(6283\Omega)} = 796 \ \mu A \]
**Power in a Inductor**

- **Instantaneous power** \((p)\) is the product of \(v\) and \(i\)
- **True power** \((P_{\text{true}})\) is zero, since no net energy is lost due to conversion to heat in the inductor
Power in a Inductor

The rate at which an inductor stores or returns energy is called **reactive power** \((P_r)\); units: \((\text{VAR})\)

\[
P_r = I_{\text{rms}} V_{\text{rms}}
\]

\[
P_r = \frac{V_{\text{rms}}^2}{X_L}
\]

\[
P_r = I_{\text{rms}}^2 X_L
\]

These equations are of the same form as those introduced in Chapter 3 for true power in a resistor
Quality Factor (Q) of a Coil

The **quality factor** (Q) is the ratio of the reactive power in the inductor to the true power in the winding resistance of the coil or the resistance in series with the coil

\[
Q = \frac{\text{reactive power}}{\text{true power}} = \frac{X_L}{R_W}
\]
Inductor Applications

- An inductor can be used in the **filter** to smooth out the ripple voltage.

![Diagram of inductor application](image-url)
Summary

- Self-inductance is a measure of a coil’s ability to establish an induced voltage as a result of a change in its current.
- Inductance is directly proportional to the square of the number of turns, the permeability, and the cross sectional area of the core. It is inversely proportional to the length of the core.
- An inductor opposes a change in its own current.
Summary

Faraday’s law states that relative motion between a magnetic field and a coil induces voltage across the coil.

Lenz’s law states that the polarity of induced voltage is such that the resulting induced current is in a direction that opposes the change in the magnetic field that produced it.
Summary

- The permeability of a core material is an indication of the ability of the material to establish a magnetic field.
- The time constant for a series RL circuit is the inductance divided by the resistance.
- In an RL circuit, the voltage and current in an energizing or de-energizing inductor make a 63% change during each time-constant interval.
Summary

- Inductors add in series
- Total parallel inductance is less than that of the smallest inductor in parallel
- Current lags voltage by 90° in an inductor
- Inductive reactance \( (X_L) \) is directly proportional to frequency and inductance
- An inductor blocks high-frequency ac