Chapter 5
Parallel Circuits
Objectives

- Identify a parallel circuit
- Determine the voltage across each parallel branch
- Apply Kirchhoff’s current law
- Determine total parallel resistance
- Apply Ohm’s law in a parallel circuit
- Use a parallel circuit as a current divider
- Determine power in a parallel circuit
A parallel circuit is one that has more than one branch.
Voltage in Parallel Circuits

The voltage across any given branch of a parallel circuit is equal to the voltage across each of the other branches in parallel.
Voltage in Parallel Circuits

Example: Determine the voltage across each resistor

$V_1 = V_2 = V_3 = V_4 = V_5 = 25 \text{ V}$
Kirchhoff’s Current Law (KCL)

The sum of the currents into a node is equal to the sum of the currents out of that node

\[ I_{IN(1)} + I_{IN(2)} + \ldots + I_{IN(n)} = I_{OUT(1)} + I_{OUT(2)} + \ldots + I_{OUT(m)} \]
Generalized Circuit Node
Illustrating KCL

\[ I_{IN(1)} + I_{IN(2)} + \ldots + I_{IN(n)} = I_{OUT(1)} + I_{OUT(2)} + \ldots + I_{OUT(m)} \]

or

\[ I_{IN(1)} + I_{IN(2)} + \ldots + I_{IN(n)} - I_{OUT(1)} - I_{OUT(2)} - \ldots - I_{OUT(m)} = 0 \]
Kirchhoff’s Current Law

Example: Determine the current through $R_2$

$I_2 = 100 - 30 - 20 = 50 \text{ mA}$
Total Parallel Resistance

The total resistance of a parallel circuit is always less than the value of the smallest resistor

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}$$
Total Parallel Resistance

**Example**: Calculate the total parallel resistance

\[
R_T = \frac{1}{R_1 || R_2 || R_3}
\]

\[
\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}
\]

\[
R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}
\]

\[
R_T = 1/((1/100) + (1/47) + (1/22)) = 13.0 \, \Omega
\]
Application of a Parallel Circuit

One advantage of a parallel circuit over a series circuit is that when one branch opens, the other branches are not affected.
Application of a Parallel Circuit

- All lights and appliances in a home are wired in parallel
- The switches are located in series with the lights

(a) Simplified diagram of room wiring
Ohm’s Law in Parallel Circuits

\[ V = IR, \ I = \frac{V}{R}, \text{ and } R = \frac{V}{I} \]

**Example**: Find the each current in below circuit

\[ R_T = \frac{1}{\left(\frac{1}{100} + \frac{1}{56}\right)} \]
\[ = 35.9 \Omega \]

\[ I_T = \frac{V_S}{R_T} \]
\[ = \frac{10}{35.9} = 279 \text{ mA} \]

\[ I_{R1} = \frac{10}{100} = 100 \text{ mA} \]
\[ I_{R2} = \frac{10}{56} = 179 \text{ mA} \]
Current Dividers

A parallel circuit acts as a **current divider** because the current entering the junction of parallel branches “divides” up into several individual branch currents.
Current-Divider Formulas for Two Branches

- The total current divides among parallel resistors into currents with values inversely proportional to the resistance values

\[ R_T = R_1 \ || \ R_2 \Rightarrow R_T = \frac{R_1R_2}{R_1 + R_2} \]

\[ I_1R_1 = I_T R_T = \frac{I_T R_1 R_2}{R_1 + R_2} \]

\[ \begin{align*}
I_1 &= \frac{R_2}{R_1 + R_2} (I_T) \\
I_2 &= \frac{R_1}{R_1 + R_2} (I_T)
\end{align*} \]
Current-Divider Formulas for Two Branches

Example: Find the each current in below circuit

\[ I_1 = 10 \left( \frac{1000}{100+1000} \right) \]
\[ = 9.09 \text{ mA} \]

\[ I_2 = 10 \left( \frac{100}{100+1000} \right) \]
\[ = 0.909 \text{ mA} \]
General Current-Divider Formula

General current-divider formula for any number of parallel branches

\[ I_x = \frac{R_T}{R_X} (I_T) \]
Power in Parallel Circuits

Total **power** in a parallel circuit is found by adding up the powers of all the individual resistors, the same as for series circuits

\[ P_T = P_1 + P_2 + \ldots + P_n \]

\[ P_T = IV_S \]

\[ P_T = I^2 R_T \]

\[ P_T = \frac{V_S^2}{R_T} \]

\[ R_T = \frac{1}{(1/68 + 1/33 + 1/22)} = 11.1 \, \Omega \]

\[ P_T = (2)^2(11.1) = 44.4 \, \text{W} \]
Open Branches

When an open circuit occurs in a parallel branch, the **total resistance increases**, the **total current decreases**, and the same current continues through each of the remaining parallel paths.

![Diagram showing closed and open branches](image)
Open Branches

When a lamp filament opens, total current decreases, the other branch currents remain unchanged.
Summary

- A parallel circuit provides more than one path for current.
- The total parallel resistance is less than the lowest-value parallel resistor.
- The voltages across all branches of a parallel circuit are the same.
- Kirchhoff’s Current Law: The sum of the currents into a node equals the sum of the currents out of the node.
- Kirchhoff’s Current Law may also be stated as: The algebraic sum of all the currents entering and leaving a node is zero.
Summary

- A parallel circuit is a current divider
- The total power in a parallel-resistive circuit is the sum of all the individual powers of the resistors making up the parallel circuit
- If one of the branches of a parallel circuit opens, the total resistance increases, and therefore the total current decreases
- If a branch of a parallel circuit opens, there is no change in current through the remaining branches