Optimal design of Fresnel lens for a reading light system with multiple light sources using three-layered Hierarchical Genetic Algorithm

Wen-Gong Chen, and Chii-Maw Uang
Department of Electrical Engineering, I–Shou University, Kaohsiung 840, Taiwan, R.O.C.

ABSTRACT
A typical Fresnel lens is not suitable to a reading light system with multiple light sources since it is designed in such a way that each groove is at a slightly different angle from the next but with the same focal length. Therefore, a Fresnel lens with suitably designed groove angles is needed for this kind of light system. In this paper, a more efficient three-layered Hierarchical Genetic Algorithm (3LHGA) is proposed to find an optimal set of groove angles for a designed Fresnel lens to optimize both the illuminance and uniformity for a reading light system with multiple light sources. The groove angles of a designed Fresnel lens are directly derived from a Fresnel lens database by two layers of control genes in the proposed 3LHGA. The proposed 3LHGA not only makes it possible to evolve a lot of groove angles as parametric genes but also further improve the performance of a Fresnel lens and increase the speed of evolution. From the simulation results we can demonstrate that the designed Fresnel lens indeed offers improvement of light-guiding performance for a multiple-LED reading light system.

Keywords: Reading light system, Light Emitting Diode (LED), Fresnel lens, Genetic Algorithms (GAs). Hierarchical Genetic Algorithms (HGAs)

1. INTRODUCTION
A typical Fresnel lens consists of a series of small, narrow concentric grooves on the surface of a lightweight plastic sheet shown in Fig. 1. Its cross section with 330 grooves is shown in Fig. 2. Since it can be made thin, flat, and at low cost, it is getting more and more popular in many applications such as lighthouses, overhead projectors, file Fresnel lenses, TV projections, condenser systems, automobile headlights, solar energy, rear projections, passive motion detectors, traffic signs, solar concentrators, collimators, and LED applications. Take LED applications for example, LED-based reading light systems may be seen everywhere nowadays. However, these systems are usually equipped with multiple LEDs as multiple light sources due to the fact that a single LED does not provide enough flux. To reading light systems with multiple light sources, a typical Fresnel lens becomes not so effective on both

* Wen-Gong Chen. E-mail: wgchen93@yahoo.com.tw. Tel.: +886-7-6577711 Ext. 6675, Fax.: +886-7-3927393.
illuminance and uniformity because its every groove is designed as a slightly different angle from the next but with the same focal length. In other words, it neither efficiently guides light rays from light sources to the reading surface, nor uniformly distributes them on the reading surface. To deal with the problem given above, one way is to design a particular Fresnel lens to fit a light system with multiple light sources, i.e. searching for a special set of groove angles for a Fresnel lens. However, it seems to be not an easy task because the searching space may become very large if the number of Fresnel lens’s grooves is big and the range of a groove’s angles is wide. Take a Fresnel lens with 330 grooves as an example, the searching space will contain \(330^{330}\) possible solutions if each designed groove contains 330 angles ranging from 0.12 degrees to 47.37 degrees. Therefore, how to find out a solution as an optimal set of groove angles from such an enormous searching space is accurately a challenging task.

In the previous work, we proposed a two-layered Hierarchical Genetic Algorithm to solve the problem of an enormous searching space. The first layer of a chromosome consisted of a series of control genes. It played an important role to reduce the searching space since the number of its control genes is a lot less than that of designed grooves. The degree of reduction depended on how grooves were translated into segments. As for the second layer, it consisted of a series of parametric genes to keep a set of chosen groove angles for evaluating a chromosome’s fitness in population. Although that work indeed made the design of a large scale of Fresnel lens used in the multiple-LED reading light system feasible, two improvements are expected to improve the performance of that lens. One is varied-length segments instead of original fixed-length segments when grooves are translated into segments. To any new Fresnel lens to be designed, it is not necessary to spend much time trying to find better segmentation any more. In fact, finding a better segmentation is difficult if there are no efficient ways are applied. The other one is increasing the variation of groove angles’ sequence in a segment. In previous work, due to the reason that a segment’s groove angles are directly picked from a Fresnel lens database according to the value of a control gene, they are in ascending or descending order sequence. In other words, the groove angles of a segment will be arranged in the designed Fresnel lens with the same sequence as they are originally stored in the Fresnel lens database. It is clear that such an arrangement of groove angles is in shortage of variation. Lacking variation may result in bad influences not only on the performances of a designed Fresnel lens in illuminance and uniformity but also on the
speed of the developed algorithm’s convergence. To make matters worse, the bigger the segment is, the greater the influence will be.

To reach two expectations mentioned above, we propose a three-layered Hierarchical Genetic Algorithm (3LHGA) in this paper. By adding a control layer to the previous approach, 2LHGA, the original segments can be divided into varied-length sub-segments and the arrangements of groove angles in an original segment can also be modified to varied arrangements in terms of a cycling mechanism. In order to compare with the previous work, a simulated reading light system in this approach is identical to that of the previous work, as well as the performance index. Therefore, it is necessary to re-describe the simulated reading light system and the performance index in this paper.

The remainder of this paper is organized as follows: Section 2 describes a simulated multiple-LED reading light system in advance. Section 3 defines the performance index used in the proposed 3LHGA. In section 4, introduction to 3LHGA and how to implement 3LHGA are presented. The simulated results are presented in section 5. Finally, we draw our conclusions in section 6.

2. A SIMULATED MULTIPLE-LED READING LIGHT SYSTEM

A simulated LED-based reading light system shown in Fig. 3 consists of three coaxially placed parts: an LED light source set, a designed Fresnel lens, and a circular reading surface. Each simulated part is constructed at the beginning of the developed programs by applying TRACEPRO\textsuperscript{7} macros. Its detailed configurations are described as follows.

Fig. 3. A simulated reading light system. Fig. 4(a). A cone-frustum shaped reflector and LED light source. Fig. 4(b). The arrangement of five LEDs in light source set.

1) The light source set owns five identical LEDs and each of which is embedded in a reflector. The reflector has the shape of a frustum of a cone with a view angle of 114.62 degrees. It has an altitude of 9 mm and a circular ceiling with a radius of 3.25 mm. Also, its inner wall is assumed to have the perfect mirror surface property. An LED point light source is placed at the center of the ceiling of the cone-frustum shaped reflector. The configuration of the combination of the cone-frustum shaped reflector and LED point light source is enlarged and
shown in Fig. 4(a). This configuration ensures that none of the light rays emitted from an LED escapes upwards. The arrangement of five LEDs in light source set is symmetrical as shown in Fig. 4(b).

2) A Fresnel lens to be designed is put at the floor of the cone-frustum shaped reflector and used to guide light rays from the light sources to the reading surface. It is a piece of lightweight plastic sheet with a radius of 165 mm and a height of 0.5 mm and located between the LED light sources and the reading surface. More precisely, its top is about 107.25 mm from LED light source and 761.5 mm from the reading surface. The topside of the plastic sheet will be grooved as ridged shapes with a series of $N_g$ (in table 1) small narrow concentric rings with same width of 0.5 mm. The ridged shapes form groove angles they are acted as the design parameters in this paper.

3) The reading surface is constructed with a thin cylinder with a radius of 495 mm and a height of 1 mm and placed below the Fresnel lens with a distance of 761.5 mm. Its topside is simply designed to have property of perfect absorption.

In order to ensure that the light rays emitted from LED light sources all pass through the designed Fresnel lens, a bigger cone-frustum shaped reflector is added to enclose five LED light sources. The light source set of five LEDs is placed at the ceiling of this reflector while a designed Fresnel lens is placed at its base.

As a whole, the constructed simulated reading light system leaves a Fresnel lens with a large set of groove angles to be designed. The most important thing to think about is how to efficiently design a set of groove angles to let more light rays emitted from LEDs light sources arrive at the reading surface and uniformly distribute over the reading surface. Before the design of such an optimal Fresnel lens, we first define an appropriate performance index as an evolution basis for the developed algorithm in the following section.

3. PERFORMANCE INDEX

Since the illuminance and uniformity are equally important in a reading light system, we have to take them into account simultaneously during the design of a Fresnel lens. Hence, although the design goal considered in this paper is to search for a set of variable groove angles to improve the performance of a designed Fresnel lens, the performance index of this approach is basically same as the previous approach\textsuperscript{12} because two approaches are developed in the same simulated system. In such a way, it will be more significant to compare the simulated results of the two developed approaches. The performance indexes are re-described below.

For simplicity, we assume each LED is a point light source and emits $N$ light rays uniformly distributed over a $2\pi$ steradians of solid angle. Due to the fact that some of the light rays do not arrive at the reading surface, the illuminance performance of a reading light system can be measured according to the number of light rays incident to the reading surface. As for the distribution uniformity of light rays over a reading surface, a good reading light system requires that the incident light rays have a specified distribution over the reading surface. For a circular reading surface, the illuminance is strongest in the center area and is gradually reduced in the outward direction.

In order to measure the actual illuminance and uniformity of distribution, the circular reading surface is divided into $N_r$ equal-area rings and each ring is further divided into $N_s$ equal-area sectors, as shown in Fig. 5. Let
\( R_s \) indicate the number of light rays incident to the \( s^{th} \) sector of the \( r^{th} \) ring. Then, the total number of light rays incident to the reading surface through a designed Fresnel lens is given by
\[
R_d = \sum_{r=1}^{N_r} \sum_{s=1}^{N_s} R_{rs},
\]
and the averaged number of light rays incident to a sector through a designed Fresnel lens is given by
\[
R_a = \frac{R_d}{N_r \times N_s}.
\]

Now, in order to take the illuminance and uniformity into account simultaneously, we define the performance index \( I \) to be maximized as follows:
\[
I = G - L,
\]
where
\[
G = (R_d - R_t) \times G_w,
\]
\[
L = \sum_{r=1}^{N_r} \sum_{s=1}^{N_s} L_{rs},
\]
Eq. (4) stands for the gain of performance index whereas Eq. (5) stands for the loss of performance index. In Eq. (4), \( R_t \) denotes the number of incident light rays to the reading surface through a typical Fresnel lens in the reading light system and \( G_w \) denotes the gain weight. The reason of deducting \( R_t \) from \( R_d \) is that a typical Fresnel lens is used as an initial condition in this approach. In Eq. (5), \( L_{rs} \) denotes the loss of performance index of the \( s^{th} \) sector of the \( r^{th} \) ring.

It is used to quantify the penalty of non-uniformity distribution for a sector that has too few or too many incident light rays and expressed as follows:
\[
L_{rs} = \begin{cases} 
\frac{R_s}{R_a} \times L_w & \text{for } R_{rs} \geq R_a \\
(1 - \frac{R_s}{R_a}) \times L_w & \text{otherwise}
\end{cases}
\]
where \( L_w \) denotes the loss weight.

Fig. 5. A reading surface with 5 equal-area rings, 120 equal-area sectors

Fig. 6 Fixed-length division mechanism
In this approach, the design goal is to search for a better set of groove angles for a Fresnel lens to make the performance index better. Therefore, how to improve the design of a Fresnel lens to reach the design goal is the point of this approach. The developed 3LHGA is described in the following section.

4. THE PROPOSED APPROACH

According to the design concept of the previous work\textsuperscript{12}, the segmentation of grooves played an important role in reducing the searching space. However, the grooves in a Fresnel lens database were segmented into fixed-length segments there. In addition, the sequence of groove angles in a segment is in the same sequence as they originally appeared in a Fresnel lens database. In such a design, the segmentation of grooves and the sequence of groove angles in a segment lack variety and probably restrict the performance of a designed Fresnel lens. To solve the problem, we propose a 3LHGA by adding a control layer to handle the problem of variation. In 3LHGA, the added control layer is acted as the second control layer and used to store the lengths of sub-segments that are divided from a segment. The segment to be divided may be a fixed-length segment or a varied-length segment. For a fixed-length segment, the division is simpler but less variable, as shown in Fig. 6. It is called the fixed-length division. For this way of division mechanism, segments divided into sub-segments are all fixed-lengths. On the other hand, for a varied-length segment, the division is a bit complicated but more variable, as shown in Fig. 7. It is an idea of composing some fixed-length segments into a bigger segment first and then decomposing it into some varied-length segments. How many fixed-length segments are composed and how many varied-length segments are decomposed is problem-dependent. For simplicity, three segments are chosen as a composing and decomposing unit. In this way, all segments divided into sub-segments are possibly varied-lengths. We call the latter way the varied-length division. For this way of division mechanism, segments divided into sub-segments are possibly all varied-length. No matter which way is used, we can reach the goal of making groove angles more variable.

![Fig. 7 Varied-length division mechanism](image1)

![Fig. 8 A cycling loading mechanism](image2)

In order to further increase the variation of designed groove angles, we adopt a cycling loading mechanism to put groove angles into parametric genes according to the values of the second layer’s control genes. It is such a way that grooves angles represented by sub-segments are put into parametric genes with cycling sequence of sub-segments themselves. So, their original sequence of those groove angles in Fresnel lens database can, therefore, be broken. For example, suppose that a segment is divided into three varied-length sub-segments, the groove angles represented by that segment will be put into parametric genes by (a) the order of the first sub-segment, the second sub-segment, and the third sub-segment, (b) the order of the second sub-segment, the third sub-segment, and the first
sub-segment, (c) the order of the third sub-segment, the first sub-segment, and the second sub-segment depending on the value of a random number. It is shown in Fig. 8. As for how many sub-segments a segment will be divided, depending on how big a segment is.

4.1 The structure of chromosome in 3LHGA

The chromosome of 3LHGA consists of three layers, two layers of control genes and on layer of parametric genes. Its structure is shown in Fig. 9. In Fig. 9, the variables $S_{i1}$ represents the length of the first sub-segment, $S_{i2}$ represents the length of the second sub-segment, etc. In the first layer, the control genes are used to represent the segment number and coded as integers, each ranging from 1 to $N_{seg}$. $N_{seg}$ denotes the number of segments and has the number of 33 in this paper. Here, a segment is a unit and includes a small group of grooves in Fresnel lens database. For instance, a segment contains 10 grooves if $N_{seg}$ is set to be 33. The distribution of 330 grooves on 33 segments and their corresponding angles are shown in Table 2. In the second layer, the control genes are used to represent the lengths of sub-segments and also coded as integers, each ranging from 0 to $L_{sub-seg}$. $L_{sub-seg}$ denotes the length of a sub-segment and has the number of 10. The lengths of sub-segments are obtained from two kinds of division mechanism described above. If $N_{sub-seg}=3$, the number of control genes in the second control layer is three times of $N_{seg}$. As for the parametric genes in the third layer, they are used to represent the groove angles derived from Fresnel lens database according to the values of control genes in the second layer. In order to increase the variety of groove angles’ sequences in the parametric genes, a described cycling loading mechanism is applied. Take $N_{seg}=33$ as an example and assume that fixed-length-segments-division is applied, if the value of the first control gene in the first layer is 3 and the values of the first three control genes in the second control layer are 3, 3, and 4, respectively, the ten groove angles in the third segment of Table 2 are extracted and put into parametric genes according to previous cycling loading mechanism to decide which sub-segment comes first, which one next, and so forth. A special structure of chromosome for this example is shown in Fig. 10. The figures in Fig. 10 are taken to two decimal places.

Table 1. Parameters used in the proposed approach

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size, $N_{p}$</td>
<td>30</td>
</tr>
<tr>
<td>Number of Groove angles, $N_{G}$</td>
<td>330</td>
</tr>
<tr>
<td>Number of light rays emitted from an LED, $N_{L}$</td>
<td>1000</td>
</tr>
<tr>
<td>Number of segments of 330 groove angles, $N_{seg}$</td>
<td>33</td>
</tr>
<tr>
<td>Mutation Rates, $M_{u}$</td>
<td>0.06</td>
</tr>
<tr>
<td>Number of equal-area rings of the reading surface, $N_{r}$</td>
<td>5</td>
</tr>
<tr>
<td>Number of equal-area sectors of a ring, $N_{s}$</td>
<td>24</td>
</tr>
<tr>
<td>Gain weight of performance index, $G_{w}$</td>
<td>10</td>
</tr>
<tr>
<td>Loss weight of performance index, $L_{w}$</td>
<td>40</td>
</tr>
<tr>
<td>Length of a sub-segment, $L_{sub-seg}$</td>
<td>10</td>
</tr>
<tr>
<td>Number of sub-segment in a segment, $N_{sub-seg}$</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 2. Distribution of 330 groove angles on 33 segments for $N_{seg}=33$

<table>
<thead>
<tr>
<th>Segment</th>
<th>Groove angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.12,0.33,0.53,0.73,0.94,1.14,1.34,1.55,1.75,1.97</td>
</tr>
<tr>
<td>2</td>
<td>3.17,3.37,3.57,3.78,3.98,2.16,2.36,2.56,2.76,2.97</td>
</tr>
<tr>
<td>3</td>
<td>4.18,4.38,4.58,4.78,4.98,5.18,5.38,5.58,5.78,5.98</td>
</tr>
<tr>
<td>31</td>
<td>44.84,44.93,45.02,45.10,45.19,45.28,45.37,45.45,45.54,45.63,45.71</td>
</tr>
<tr>
<td>32</td>
<td>45.80,45.89,45.97,46.05,46.14,46.22,46.30,46.39,46.47,46.55</td>
</tr>
<tr>
<td>33</td>
<td>46.64,46.72,46.80,46.88,46.96,47.04,47.13,47.21,47.29,47.37</td>
</tr>
</tbody>
</table>
4.2 Implementation of 3LHGA

In actual implementation of 3LHGA, an initial population with $N_p$ chromosomes is generated first. The values of control genes of the first layer in every chromosome are set from $I$ to $N_{seg}$ in sequence to let parametric genes start with groove angles as the typical Fresnel lens database. In order to increase the variety of groove angles’ sequences in a designed Fresnel lens, the groove angles in database are extracted through two segment division mechanisms and a cycling loading mechanism. In fact, segment division mechanisms are mainly used to create the control genes of the second layer. It is easily known that different division ways produce different varieties of groove angles’ sequence in parametric genes. No matter which one is chosen, it can help to create groove angles with more variety. The cycling mechanism for creating parametric genes is also having similar effect as segment division mechanisms. Those effects can be verified by observing every chromosome’s fitness in practical implementation. After generation of an initial population, fitness values of chromosomes are computed according to performance index defined in Eq. (3). Then, all chromosomes are sorted according to their fitness values. The sorted chromosomes $C_i$ and $C_{i+1}$ have the property $F(C_i)\geq F(C_{i+1})$ for $i=1,2,\ldots,N_p$, where $F(C)$ represents the fitness value of chromosome $C$.

With the initial sorted chromosomes, the offspring chromosomes are generated through first mutation operation and then crossover operation generation by generation. The mutation operation with a mutation rate $M_r$ is only performed on the first layer’s control genes of each chromosome. In a chromosome $C_i$, let $g_{ij}$ denote the $j^{th}$ control gene of the first layer and $g_{ik}$ and $g_{i(k+2)}$ denote the $k^{th}$, $(k+1)^{th}$, and $(k+2)^{th}$ control genes of the second layer, where $k=(j-1)*3+l$. For the control gene $g_{ij}$, we generate a random numbers $r$. If $r$ is less than $M_r$, then the control gene $g_{ij}$ is mutated to a new one called $g_{ij}'$ which has a value between $I$ and $N_{seg}$, otherwise control gene is not mutated, i.e., $g_{ij}' = g_{ij}$. If the control gene $g_0$ is mutated to $g_{ij}'$, the corresponding three control genes of the second layer in a chromosome are updated with three new numbers to represent three new sub-segments, i.e., $g_{ik}$ is...
replaced with \( g_{ik} \), \( g_{(k+1)} \), and \( g_{(k+2)} \) is replaced with \( g'_{i(k+1)} \), \( g'_{i(k+1)} \), and \( g'_{i(k+2)} \) represent the lengths of newly divided sub-segments. Otherwise, \( g_{ik} \), \( g_{(k+1)} \) and \( g_{(k+2)} \) are not changed, i.e., \( g_{ik} = g_{ik} \), \( g_{i(k+1)} = g_{i(k+1)} \), and \( g_{i(k+2)} = g_{i(k+2)} \). A mutated chromosome \( C'_i \) is formed from the first layer’s control genes \( g_{ij} \), \( j=1, 2, 3, \ldots, N_{seg} \), the second layer’s control genes \( g_{i(k+1)} \) and \( g'_{i(k+2)}, k=(j-1)*3+1 \), and the parametric genes through the cycling loading mechanism. The chromosome \( C_i \) is replaced by the chromosome \( C'_i \) if the mutated chromosome \( C'_i \) has a higher fitness value than \( C_i \). Having performed the mutation operations on all chromosomes of the current generation, we make crossover operation to further improve the fitness of chromosomes of the population. The crossover operation is also merely performed on control genes for all chromosome pairs \( (C_i, C_{i+N_p/2}), i=1,2,3,\ldots,N_p/2 \). For a given chromosome pair \( (C_i, C_{i+N_p/2}) \), we generate three different random numbers \( r_1, r_2, \) and \( r_3 \) over the interval \((0,1)\) and obtain two integers \( k_1 = \lfloor N_{seg} \times r_1 \rfloor \) and \( k_2 = \lfloor N_{seg} \times r_2 \rfloor \), which represent the crossover points and lie in the range \([2,N_{seg}-1]\). Assume that \( k_1 \) is less than \( k_2 \). In the chromosome \( C_i \), let \( C_{i(a:b)} \) denote a control gene section from the \( a^{th} \) control gene to the \( b^{th} \) control gene in the first layer and \( C_{j(a':b')} \) denote a control gene section from the \( c^{th} \) control gene to the \( d^{th} \) control gene in the second layer. Then, the first layer’s control genes of offspring chromosomes generated by crossover are given by

\[
C'_{i(1:N_{seg})} = C_{i(1:k_1)} + C_{j(k_1:k_2)} + C_{j(k_2:N_{seg})},
\]

and

\[
C'_{j(1:N_{seg})} = C_{j(1:k_2)} + C_{i(k_2:k_1)} + C_{i(k_1:N_{seg})}.
\]

Let \( x \) and \( y \) denote the beginning and end position of the second layer’s control genes in a chromosome, respectively, i.e., \( x=N_{seg}+1 \), \( y=N_{seg}+3*N_{seg} \). The second layer’s control genes of offspring chromosomes generated by crossover are given by

\[
C'_{i(x:y)} = C_{i(x+3(k_1-1):x+3(k_2-1))} + C_{j(x+3(k_1-1):x+3(k_2-1))} + C_{i(x+3(k_1-1):x+3(k_2-1))},
\]

and

\[
C'_{j(x:y)} = C_{j(x+3(k_2-1):x+3(k_1-1))} + C_{i(x+3(k_2-1):x+3(k_1-1))} + C_{j(x+3(k_2-1):x+3(k_1-1))}
\]

if \( r_3 \) is less than or equal to 0.5, otherwise by

\[
\text{otherwise by}
\]
\[ C_{i}(1;N_{new}) = C_{j}(k_{1}) + C_{i}(k_{2}) + C_{j}(k_{3};N_{new}) \]  
\[ C_{j}(1;N_{new}) = C_{i}(k_{1}) + C_{j}(k_{2}) + C_{i}(k_{3};N_{new}) \]  
\[ C_{i}^{*}(x,y) = C_{j}(x + 3y(k_{1} - 1)) + C_{i}(x + 3y(k_{2} - 1)) + C_{j}(x + 3y(k_{3} - 1)) + C_{j}(x + 3y(k_{4} - 1)) \]  
and
\[ C_{j}^{*}(x,y) = C_{i}(x + 3y(k_{1} - 1)) + C_{j}(x + 3y(k_{2} - 1)) + C_{i}(x + 3y(k_{3} - 1)) + C_{j}(x + 3y(k_{4} - 1)) \]

If varied-length division mechanism is adopted, the neighboring control genes of \( k_{1} \) and \( k_{2} \) are needed to be divided again. Finally, the parametric genes of \( C_{i}^{*} \) and \( C_{j}^{*} \) are obtained through the cycling loading mechanism. The control genes of the parent chromosome \( C_{i} \), are replaced by the control genes of the offspring chromosome \( C_{j}^{*} \), if \( C_{i}^{*} \) has a higher fitness value than \( C_{i} \). \( C_{j} \) and \( C_{j}^{*} \) follow the same replacement policy as \( C_{i} \) and \( C_{i}^{*} \). After finishing the crossover operations, we obtain a new generation of chromosomes, which have higher fitness values than the previous generation. By iterating the above iterations, an evolution keeps going on until a specified number of generations have been performed.

Finally, the detail steps of our proposed approach are drawn as a flow diagram shown in Fig. 11 and the simulation results will be described and discussed in the follow section.

![Fig. 11 A block diagram of 3LHGA](image1)

![Fig. 12 Convergence statuses for FL-3LHGA, FLSD and FL-2LHGA, FLSD.](image2)
5. SIMULATION RESULTS

We have proposed a three-layered Hierarchical Genetic Algorithm to increase the variation of grooves angles in a designed Fresnel lens. There are two kinds of segment divisions to create different variations. They have been described before and shown in Fig. 6 and 7, respectively. The division way in Fig. 6 is called fixed-length division (FLD) and the other one is called varied-length division (VLD). No matter which division, the size of a segment plays an important role to influence the performance of a designed Fresnel lens finally. However, it belongs to a NP-hard decision problem. In the sequel, the designed Fresnel lens is designated as FL_3LHGA_33_FLD if it is designed by the approach of 3LHGA with a segment size of 33 and the division way of FLD, FL_3LHGA_33_VLD if same approach and segment size but the division way is changed by VLD, FL_2LHGA_33_FLD if 3LHGA is changed by 2LHGA.

There are two experiments presented below and the parameters used in this approach are listed in Table 1.

For the first experiment, we basically intend to present the performance of 3LHGA compared to 2LHGA for the reading light system simulated in section 2. For simplicity, we just take $N_{seg} = 33$ as an example and use fixed-length segment division. The experimental results are shown in Fig. 12, 13, 14. From Fig. 12, it is clear that 3LHGA converges faster and has better fitness within a long period of evolutions. Since fitness is evaluated by considering both illuminance and uniformity, better fitness means better performance. Compare Fig. 13 to Fig. 14, it is more obvious that 3LHGA indeed works better than 2LHGA since more light rays are directed into the reading surface. They are 4738 and 4631 shown at the footnotes in Fig. 13 and Fig 14, respectively. It is obvious that the illumination of reading light system with 3LHGA has increased by 2.31% as compared with that of the reading light system with 2LHGA. The cross section of FL_3LHGA_33_FLD is shown in Fig. 15.
For the second experiment, the comparison of results between \( FL_{3LHGA_{33}}^{FLD} \) and \( FL_{3LHGA_{33}}^{VLD} \) is shown in Fig. 16. The blue line denotes the convergence status of \( FL_{3LHGA_{33}}^{FLD} \) and the black line denotes the status of \( FL_{3LHGA_{33}}^{VLD} \). From their convergence trends, it is clear that \( FL_{3LHGA_{33}}^{VLD} \) will create a better solution than \( FL_{3LHGA_{33}}^{FLD} \) if sufficient evolutions are performed. It stands to reason that the convergence speed of \( FL_{3LHGA_{33}}^{VLD} \) is slower than \( FL_{3LHGA_{33}}^{FLD} \) since the varieties of groove angles by \( FL_{3LHGA_{33}}^{VLD} \) is greater than \( FL_{3LHGA_{33}}^{FLD} \). Suffering from the time-consuming of evolving the developed algorithms, they are just evolved 1600 generations. Therefore, how to improve the convergence speed caused by the complicated structure and algorithm of 3LHGA with varied-length division mechanism is worth being researched in the future.

6. CONCLUSIONS

We have presented a three-layered HGA-based approach to improve a Fresnel lens for a reading light system with multiple light sources. By using the scheme of three-layered HGA, we not only solve the complexity problem of the huge searching space formed by a lot of grooves, but also increase the varieties of groove angles in a designed Fresnel lens to improve the performance compared to a two-layered HGA-based approach. However, some designed parameters are defined as default values such as the number of sub-segments and the number of composing factor when varied-length division mechanism is applied. The reasons that they are all set to be three is just for simplicity in program coding. Since these numbers play an important role in this approach, it will be better if a more efficient or expectable way to decide those numbers in the future work.

ACKNOWLEDGMENT

This work was supported in part by I-Shou University under contract ISU-93-01-03, and in part by the National Science Council under grant NSC 94-2215-E-214-005.
REFERENCES